Power Number

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The Power Number, also called the Newton Number, is a dimensionless number used in describing the power used in connection with the use of impellers in liquid filled vessels, such as bioreactors, fermenters, chemical reactors. It relates the actual power encountered in a flow system to that encountered when a moving surface impacts a fluid.

From fundamental physics, in a flowing system, the pressure, p, arising from stopping a fluid is proportional to the product of the fluid density ρ, and the square of the velocity, v.

$$p \sim \rho v^2$$

In a rotating system, the representative velocity typically used is the impeller tip velocity, which is proportional to the product of the rotational speed, $\omega = 2\pi N$ and the diameter, where N is the rotational speed in revolutions per second. Then:

$$v = D/2 \cdot \omega \sim D \cdot N$$

Then the pressure is proportional to:

$$p \sim \rho D^2 \cdot N^2$$

A force, F, is encountered when that pressure acts on an area. For an impeller, the representative area is assumed to be proportional to the diameter, D, squared, as for a circle.

$$A \sim D^2$$

and

$$F = p \cdot A \sim (\rho D^2 \cdot N^2) \cdot (D^2)$$

The power, P, used at the impeller is the product of the force and the velocity:

$$P = F \cdot v = p \cdot A \sim (\rho D^2 \cdot N^2) \cdot (D^2) \cdot (D \cdot N)$$

Or:

$$P \sim \rho D^5 \cdot N^3$$

The proportionality constant is called the Power Number, N_p. It is similar to a Drag Coefficient in aerodynamic systems (like C_d for vehicles) and is a dimensionless number. Therefore, we can define the Power Number:

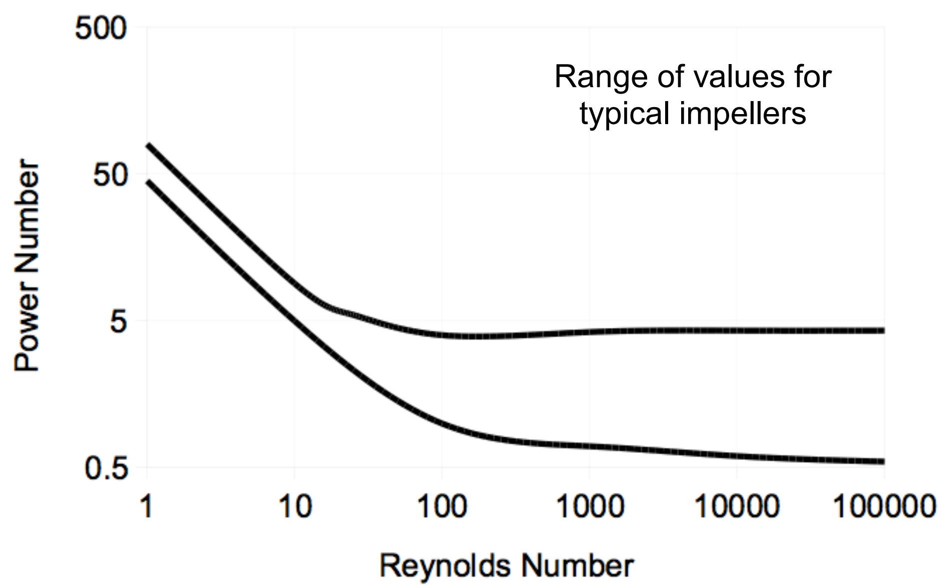
$$N_p = \frac{P}{\rho D^5 \cdot N^3}$$

In correlating power requirements in flow reactors, it is found that the Power Number is useful in combination with another dimensionless number, the Reynolds number. The Reynolds number, Re_D, is the relation between inertial forces and viscous forces and can be found to be:

$$Re_D = \frac{\rho \cdot v \cdot D}{\mu} = \frac{\rho \cdot N \cdot D^2}{\mu}$$

Where for convenience, N·D is taken as the reference velocity, though it is not the actual rotor tip velocity.

Using these two dimensionless numbers, the power can be related to the flow conditions (impeller revolutions and fluid properties) over a wide range of conditions. Typical results are shown in the following figure:



Power Number as a function of Reynolds number for a range of typical impellers.

Example:

Consider a case in the region where the Power Number is independent of the Reynolds number. Using water as an example, the density is 1,000 kg/m3, and the viscosity 0.9 cP = 0.0009 kg/m-s.

In the paper by Kaiser, et. al., reactors were measured to determine power consumption. Among those tested was a Rushton turbine, with an diameter of 53.3 mm operating at speeds between 150 and 450 rpm[1]. The Power Number was shown to be nearly constant at about 4.15. Taking revolutions of 450 rpm (450/60=7.5 rev/s), the Reynolds number is

$$\mathrm{Re}_D = \frac{\rho \cdot N \cdot D^2}{\mu} = \frac{1000 \cdot 450/60 \cdot 0.0533^2}{0.0009} = 23700$$

From the power measurements, it was found that the power needed to stir the water (excluding losses in the bearings and electrical equipment) was 0.75 W, which gives a Power Number of:

$$N_p = \frac{P}{\rho \cdot D^5 \cdot N^3} = \frac{0.75}{1000 \cdot 0.0533^5 \cdot 7.5^3} = 4.13$$

which agrees well with their N_p Re_D relationship.

Conversely, if the relationship between N_p is Re_D known, the power requirements can be determined for different conditions. Consider the above example but assume that the impeller follows the top curve in the figure shown, i. e. that the limiting Power Number is 4.90. Two fluids can be considered, water and a Sucrose solution, where the density becomes 1,285 kg/m^3 and the viscosity 0.058 kg/m-s.

For water, the Reynolds number is still 23,700, while for the Sucrose solution:

$$Re_D = \frac{\rho \cdot N \cdot D^2}{\mu} = \frac{1000 \cdot 7.5 \cdot 0.0533^2}{0.0055} = 498$$

This is a significantly lower Reynolds number, but the graph indicates that the Power Number is still unchanged. Had Reynolds number been much lower (slower speed or higher viscosity), Power Number would have increased significantly.

So, the power requirements can be calculated for each condition.

For the water:

$$P = N_p \cdot \rho \cdot D^5 \cdot N^3 = 4.9 \cdot 1000 \cdot 0.0533^5 \cdot 7.5^3 = 0.889\, W$$

For the Sucrose:

$$P = N_p \cdot \rho \cdot D^5 \cdot N^3 = 4.9 \cdot 1285 \cdot 0.0533^5 \cdot 7.5^3 = 1.143\, W$$

an increase of 28 %, due only to the density increase.

As long as the Reynolds number is significantly high, the viscosity doesn't affect the Power Number, only the density. But higher viscosity and lower revolutions could change the flow regime to a more friction dominated regime, where the Power Number is no longer constant, and viscosity could affect the Power Number.

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1 Stephan C. Kaiser, Sören Werner, Valentin Jossen, Matthias Kraume and Dieter Eibl, "Development of a method for reliable power input measurements in conventional and single-use stirred bioreactors at laboratory scale" Engineering in Life Sciences, **2017**, 17, 500-511.

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